NAG Toolbox for MATLAB

f08nn

1 Purpose

f08nn computes the eigenvalues and, optionally, the left and/or right eigenvectors for an n by n complex nonsymmetric matrix A.

2 Syntax

```
[a, w, vl, vr, info] = f08nn(jobvl, jobvr, a, 'n', n)
```

3 Description

The right eigenvector v_j of A satisfies

$$Av_j = \lambda_j v_j$$

where λ_j is the jth eigenvalue of A. The left eigenvector u_j of A satisfies

$$u_j^{\mathrm{H}} A = \lambda_j u_j^{\mathrm{H}}$$

where u_j^{H} denotes the conjugate transpose of u_j.

The matrix A is first reduced to upper Hessenberg form by means of unitary similarity transformations, and the QR algorithm is then used to further reduce the matrix to upper triangular Schur form, T, from which the eigenvalues are computed. Optionally, the eigenvectors of T are also computed and backtransformed to those of A.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvl – string**

If **jobvl** = 'N', the left eigenvectors of A are not computed.

If **jobvl** = 'V', the left eigenvectors of A are computed.

Constraint: **jobvl** = 'N' or 'V'.

2: **jobvr – string**

If **jobvr** = 'N', the right eigenvectors of **a** are not computed.

If **jobvr** = 'V', the right eigenvectors of **a** are computed.

Constraint: **jobvr** = 'N' or 'V'.

3: **a**(**lda**,$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n matrix A.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldvl, ldvr, work, lwork, rwork

5.4 Output Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

a has been overwritten.

2: **w(∗) – complex array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

Contains the computed eigenvalues.

3: **vl(ldvl,∗) – complex array**

The first dimension, **ldvl**, of the array **vl** must satisfy

if **jobvl** = 'V', **ldvl** $\geq \max(1, \mathbf{n})$;
ldvl ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobvl** = 'V', the left eigenvectors u_j are stored one after another in the columns of **vl**, in the same order as their corresponding eigenvalues.

If **jobvl** = 'N', **vl** is not referenced.

$u_j = \mathbf{vl}(:, j)$, the jth column of **vl**.

4: **vr(ldvr,∗) – complex array**

The first dimension, **ldvr**, of the array **vr** must satisfy

if **jobvr** = 'V', **ldvr** $\geq \max(1, \mathbf{n})$;
ldvr ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobvr** = 'V', the right eigenvectors v_j are stored one after another in the columns of **vr**, in the same order as their corresponding eigenvalues.

If **jobvr** = 'N', **vr** is not referenced.

$v_j = \mathbf{vr}(:, j)$, the jth column of **vr**.

5: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvl**, 2: **jobvr**, 3: **n**, 4: **a**, 5: **lda**, 6: **w**, 7: **vl**, 8: **ldvl**, 9: **vr**, 10: **ldvr**, 11: **work**, 12: **lwork**, 13: **rwork**, 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements $i+1 : \mathbf{n}$ of **w** contain eigenvalues which have converged.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* 1999 for further details.

8 Further Comments

Each eigenvector is normalized to have Euclidean norm equal to unity and the element of largest absolute value real and positive.

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is f08na.

9 Example

```
jobvl = 'No left vectors';
jobvr = 'Vectors (right)';
a  =  [complex(0,  +0),  complex(-4.11,  +3.7),  complex(-0.34,  +1.01),
complex(1.29, -0.86);
         complex(0,  +0),  complex(1.52,  -0.43),  complex(1.88,  -5.38),
complex(3.36, +0.65);
          complex(0,  +0),  complex(2.5,  +3.45),  complex(0.88,  -1.08),
complex(0.64, -1.48);
       complex(-1.1, +0.82), complex(1.81, -1.59), complex(3.25, +1.33),
complex(1.57, -3.44)];
[aOut, w, vl, vr, info] = f08nn(jobvl, jobvr, a)
```

```
aOut =
   -4.8428 + 0.1160i   -0.5654 - 2.5750i    1.3645 - 0.2794i   -0.6893 -
0.1392i
         0              -0.1019 - 0.0784i   -0.0453 - 3.6274i    0.0590 +
1.5385i
         0                    0              2.2454 - 3.1030i    2.2622 -
1.2864i
         0                    0                   0              6.6694 -
```

```
1.8845i
w =
  -4.8428 + 0.1160i
  -0.1019 - 0.0784i
   2.2454 - 3.1030i
   6.6694 - 1.8845i
vl =
   1.0e-53 *
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.2896                  0
vr =
    0.6389               0.9676               0.4172 - 0.2851i  -0.4783 +
0.1809i
   0.3594 + 0.4192i   0.0552 + 0.0759i   0.0013 + 0.3266i   0.6645
    0.2314 - 0.4052i    0.0393 - 0.1345i   -0.0528 - 0.2842i    0.2389 +
0.3280i
   -0.2469 - 0.0905i    0.1730 + 0.0738i    0.7446                0.3302 -
0.1525i
info =
           0
```